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SUBJECTIVE PROBABILITY AND DELAY

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Human subjects indicated their preference between a hypothetical $1,000 reward available with various probabilities or delays and a certain reward of variable amount available immediately. The function relating the amount of the certain-immediate reward subjectively equivalent to the delayed $1,000 reward had the same general shape (hyperbolic) as the function found by Mazur (1987) to describe pigeons' delay discounting. The function relating the certain-immediate amount of money subjectively equivalent to the probabilistic $1,000 reward was also hyperbolic, provided that the stated probability was transformed to odds against winning. In a second experiment, when human subjects chose between a delayed $1,000 reward and a probabilistic $1,000 reward, delay was proportional to the same odds-against transformation of the probability to which it was subjectively equivalent.

Key words: choice, delay, probability, discounting, humans

The idea that people behave similarly in the face of probability and delay was first proposed by Rotter (1954) and tested by Mischel (1966) in the context of "delay of gratification." According to Rotter, people choose a smaller more immediate reward over a larger but delayed reward because, in the local culture, promises of delayed reward are rarely given or, if given, broken. In other words, delays of gratification act like less-than-unity probabilities; longer delays correspond to lower probabilities.

The present studies investigated the relation between subjective probability and delay with human subjects. In a series of psychophysical tasks, we first attempted to establish separate delay and probability discount functions. A second experiment related subjective delay to probability of outcome. The word *subjective* is used here to refer to the reward judged by the *subject* to be equivalent in value to a reward stated by the experimenter. *Subjective* therefore refers to a response or a judgment of the subject and not necessarily to any part of (or representation within) the subject.

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Hypothetical rewards in human operant research. In everyday speech the word *imagination* refers to the creation of an image or picture in the mind. Modern cognitive psychologists have adopted this use to refer to internal representations of aspects of the environment. But an act of imagination also can have a behavioral meaning. A person successfully imagining a lion might show signs of panic (running away or crying out in alarm) rather than introspection or self-contemplation (closing his eyes, nodding his head and saying, "Ah, I see it," in a calm voice). Mischel, Shoda, and Rodriguez (1989) found that children who were instructed to imagine an exposed food reward as an inedible object (a pretzel as a log, for instance) waited longer for a larger food reward than did uninstructed children. However, even without specific instructions, many children "talked quietly to themselves, sang, created games with their hands and feet, and even tried to go to sleep during the waiting time" (p. 935). These children were imagining the absence of the small reward (in the behavioral sense of the term); that is, they were behaving as they would have behaved had it not been present. Instructions are, in operant terms, discriminative stimuli for complex behavioral patterns (Hayes, Brownstein, Haas, & Greenway, 1986). The effective proposition of any instruction must, in the past, have been a discriminative stimulus for real contingencies. The qualifiers "as if," "imagine," or "hypothetical" make the instructions discriminative stimuli for acts of imagination.

It is reasonable to suppose that the ability to imagine is an essential ingredient of a person's self-control, an ability that develops with age. In the experiments to be reported here, adult human subjects were asked to imagine choosing between significant amounts of money discounted by various delays and probabilities. The experimental procedures were borrowed from psychophysical methods previously found to generate replicable, functional relationships. Whether the subjects in these experiments were successful in imagining the choice alternatives may be judged by comparing their behavior to that of nonhumans choosing between significant rewards (as in typical operant choice procedures) as well as to the behavior of humans choosing in the laboratory between (necessarily) nonmeaningful rewards (the catch-22 of human laboratory research is that the more relevant a motivational variable is to everyday human life, the less ethically justifiable is the manipulation of that variable in the laboratory). In any case, the imagination of the human subjects in the present experiments (and, in general, in experiments in human decision) is both internally coherent, in the sense that the results of one experiment may be predicted from another, and externally valid, that is, consistent with the behavior of nonhumans choosing among real rewards in corresponding situations.

EXPERIMENT I

Kahneman and Tversky's (1979) prospect theory suggests the following general form for probability discount functions:

$$v_p = f(p)V \tag{1}$$

where v_p is the discounted value of a probabilistic reward, $f(p)$ is a function of the probability of the reward, and V is the undiscounted value of the reward. According to subjective expected utility theory, the standard normative theory of probabilistic choice, $f(p) = p$. (Consequently, $v_p = pV$ where pV is the *expected value* of the reward.) Many studies of probabilistic choice have disconfirmed the normative theory. For instance, Allais (1953) discovered that preference between a pair of probabilistic alternatives ($v_{p1} > v_{p2}$) may reverse when both alternatives are made conditional on a third probabilistic event (q). (It is not possible that $v_{p1} > v_{p2}$ and $qv_{p1} < qv_{p2}$.) To explain Allais' paradox as well as several other nonnormative findings in human decision experiments, Kahneman and Tversky's prospect theory proposes that $f(p)$ in Equation 1 is a complex but unspecified function of p with the following properties: $f(0) = 0; f(1) = 1$; over most of the range of p, $f(p) < p$; for very low but nonzero values of p, $f(p) > p$; the proposed function is discontinuous at the endpoints ($p = 0$, $p = 1$) but continuous otherwise.

Just as the standard normative theory of probabilistic discounting predicts consistency of preference whenever two probabilistic alternatives are further discounted by the same probability (q in the above example), so the standard normative theory of delay discounting predicts consistency of preference whenever two delayed alternatives are further discounted by the same delay (Benzion, Rapoport, & Yagil, 1989). This is why the interest paid by savings banks usually accumulates exponentially (i.e., by continuous compounding). A sum of \$100 earns the same interest over a fixed future period regardless of whether it is newly deposited or has accumulated to that amount from previous deposits. Suppose you deposited \$100 in Bank A now and in 5 years it had grown to \$250. If you left the \$250 on deposit and deposited \$200 in Bank B which paid the same exponential interest rate as Bank A there would be no time, no matter how long, where the values of the two deposits would reverse.

If, however, your money grew hyperbolically, the values *would* reverse. Rearranging a hyperbolic discount function (Equation 2a, below) to calculate growth (as opposed to decay), we can solve for the growth of an initial deposit v to a value V in d years. Suppose, on January 1, 2000, you deposit \$100 in a hyperbolic bank ($d_1 = 0$, $V_1 = v_1 = \$100$). Five years later (January 1, 2005) $d_1 = 5$ and by hypothesis $V_1 = \$250$. Solving Equation 2a, k must have been 0.3. At this point another deposit ($v_2 = \$200$) is made at the same (hyperbolic) bank ($k = 0.3$); on January 1, 2005, therefore, $d_2 = 0$ and $V_2 = v_2 = \$200$; now $V_2 < V_1$. Another 10 years pass; it is January 1, 2015. At this point $d_1 = 15$ and $v_1 = \$100$; $d_2 = 10$ and $v_2 = \$200$. Solving Equation 2a again, $V_1 = \$550$ but $V_2 = \$800$; now $V_2 > V_1$. The values reverse because, according to Equation 2a, duration (d) acts multiplicatively on the *initial* deposit (v). As the years pass, any differences in time of deposit are diminished in importance relative to differences in the amount of the initial deposit. Exponential growth, on the other hand, describes compound interest. Interest is calculated for each period based on the amount accumulated at the start of the period, not on the initial deposit. Therefore, with exponential growth, values can never reverse. Hence, deviations from exponential discount functions (e.g., towards hyperbolic discount functions) are often considered by economists to be "irrational" (Strotz, 1956).

There is indirect evidence (Ainslie, 1974; Logue, 1988; Rachlin & Green, 1972) with nonhuman and human subjects that subjective delay discount functions do indeed deviate from the exponential form. For instance, a pigeon might choose two pellets of food delayed by 14 s over one pellet delayed by 10 s (both alternatives fixed in time) but reverse its preference after 10 s has passed, choosing one pellet immediately over two pellets delayed by 4 s. This sort of preference reversal is predicted by hyperbolic discount functions but not by exponential ones.

On the basis of a series of experiments with pigeons as subjects, Mazur (1987) found direct evidence that pigeon's delay discount functions are not exponential but are hyperbolic. Mazur suggested the following hyperbolic delay discount function:

$$v_d = g(d)V \tag{2}$$

$$v_d = \frac{V}{1 + kd} \tag{2a}$$

where v_d is the discounted value of a delayed reward, d is delay between choice and reward, and k is a constant proportional to degree of discounting.

Rachlin, Logue, Gibbon, and Frankel (1986) noted that, with repeated probabilistic events, the average delay to an outcome is related to the probability of that outcome by the following waiting-time function:

$$d = \frac{t + c}{p} - t \tag{3}$$

where d is the average waiting time between choice (the beginning of the first trial) and a repeated probabilistic event, p is the probability of the event, t is the intertrial interval, and c is the trial duration. If c is small relative to t,

$$d = (t/p) - t = t[(1/p) - 1] = t\Theta \tag{3a}$$

where $\Theta = (1/p) - 1$ or "odds against." (For instance, the odds against a gamble with a probability of .1 paying off are 9:1.) With repeated gambles, odds against is the average number of losses expected before a win. As Skinner (1953) noted, the pattern of bets and wins in repeated gambles is the same as that of responses and reinforcers in a variable-ratio (VR) schedule. If the net gain or loss from a string of losses followed by a win were subjectively discounted (according to Equation 2a) by the string's duration, repeated gambles of zero or negative net expected value (expected gain minus cost of bet) could take on a positive subjective value (Rachlin, 1990). Temporal discounting (exponential as well as hyperbolic) may thus account for people's tendency to gamble even in games of negative expected value.

Rachlin et al. (1986) showed that substitution of Equation 3 or 3a into Equation 2a results in a probability discount function with most of the properties specified by prospect theory. The present experiment attempted to obtain separate probability and delay discount functions with human subjects to determine whether such a substitution is empirically justified (i.e., can anticipated delay and stated probability be interchanged in this way?). Two equations were tested as descriptions of the delay discount function. The hyperbolic function of Equation 2a was selected because it describes delay discounting of nonhumans (Mazur, 1987). An exponential discount function was also tested because exponential functions have been traditionally considered normative models for delay discounting (Strotz, 1956):

$$v_d = Ve^{-kd}. \tag{4}$$

Figure 1 illustrates the differing shapes of the functions of Equations 2a and 4 with different values of k and a unit reward ($V = 1.0$). As the parameter k in Equations 2a and 4 increases from zero, the discount curve increases in steepness.

Analogous equations were tested as descriptions of the probability discount function. A hyperbolic discount function for probabilistic outcomes is:

$$v_p = \frac{V}{1 + h\Theta} = \frac{pV}{p + h(1 - p)} \tag{5}$$

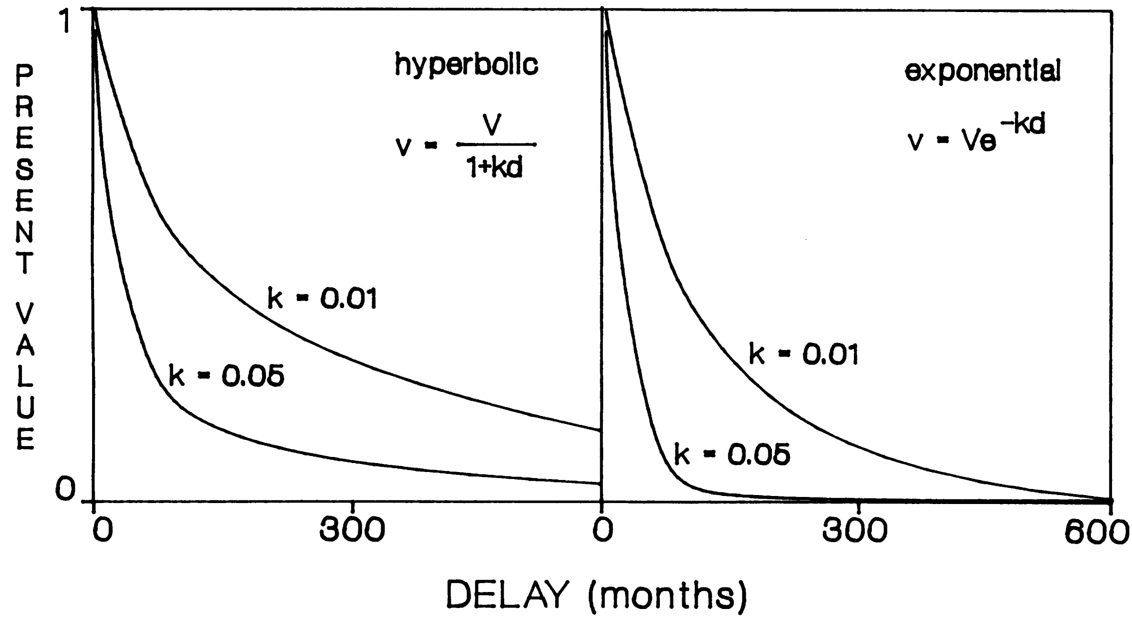

Fig. 1. Two possible delay discount functions (adapted from Mazur, 1987) with different parameter values.

where $\Theta = (1/p) - 1$. The corresponding exponential function for probabilistic discounting is:

$$v_p = Ve^{-h\Theta}. \tag{6}$$

Figure 2 illustrates the differing shapes of the functions of Equations 5 and 6 with different values of $h\Theta$. When $h = 1$, Equation 5 predicts that probabilistic outcomes will be discounted according to their expected values (expected value $= pV$). When $h > 1$, Equation 5 predicts overdiscounting of probabilistic outcomes (rel-

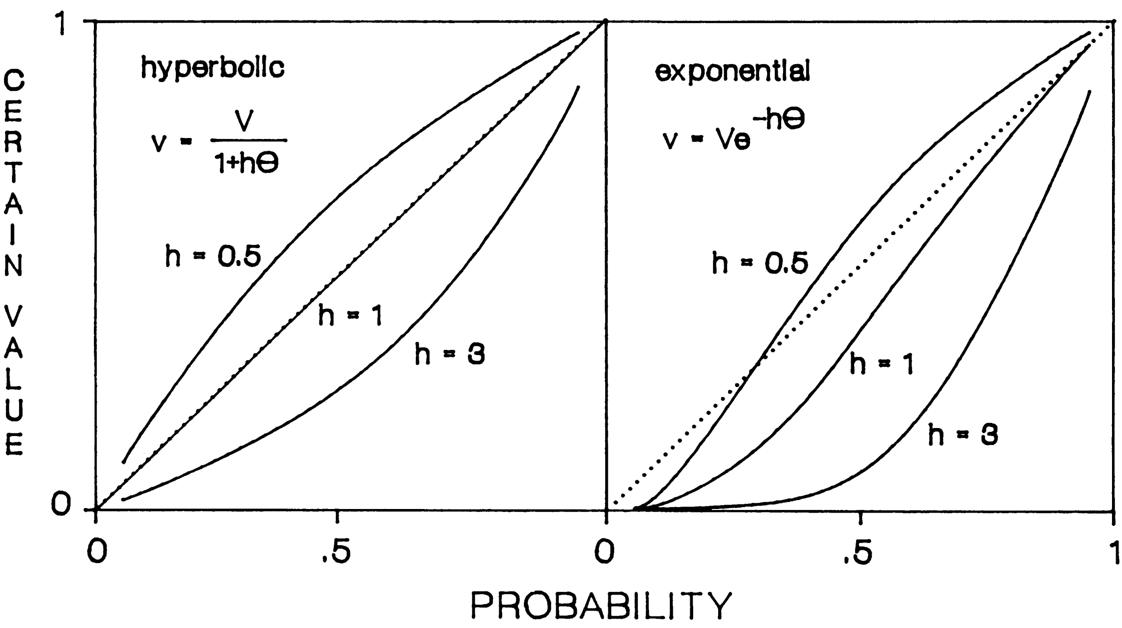

Fig. 2. Two possible probability discount functions with different parameter values.

ative to expected value), generating a function similar (over most of its range) to the one proposed by Kahneman and Tversky (1979). When $h < 1$, Equation 5 predicts underdiscounting of probabilistic outcomes.

Method

Subjects

Eighty students enrolled in an undergraduate psychology course at the State University of New York at Stony Brook served as subjects. Their participation in this experiment was a course requirement. Forty of the subjects were used to obtain a probability discount function, and 40 were used to obtain a delay discount function.

Materials

Each subject was tested individually in a small room containing two chairs and a table. Cards were presented in pairs to all subjects. One card stated an amount of money to be paid for sure ($1,000, $990, $980, $960, $940, $920, $900, $850, $800, $750, $700, $650, $600, $550, $500, $450, $400, $350, $300, $250, $200, $150, $100, $80, $60, $40, $20, $10, $5, or $1). For subjects in the probability discount group, the other card stated a probability of $1,000 as a percentage (95%, 90%, 70%, 50%, 30%, 10%, and 5% chances of winning $1,000), whereas for subjects in the delay discount group, the other card stated a delay of $1,000 (1 month, 6 months, 1 year, 5 years, 10 years, 25 years, and 50 years).

Procedure

Subjects in both probability and delay discount groups were asked to state a preference between the two cards. The probabilistic or delayed $1,000 card remained in front of the subject while the certain-immediate cards were presented one by one next to it. Subjects indicated their preference by pointing to one of the cards.

The order in which both probabilistic $1,000 and delayed $1,000 cards were presented was from highest valued to lowest valued. Thus, the highest probabilities and lowest delays were tested first. For each probability or delay, the set of certain-immediate amounts was titrated up and then down for 20 subjects and down and then up for the other 20. A subject was considered to have switched to the initially dispreferred alternative after two choices in a row of that alternative. Points of equivalence were obtained by averaging the amounts just before and just after the switch.

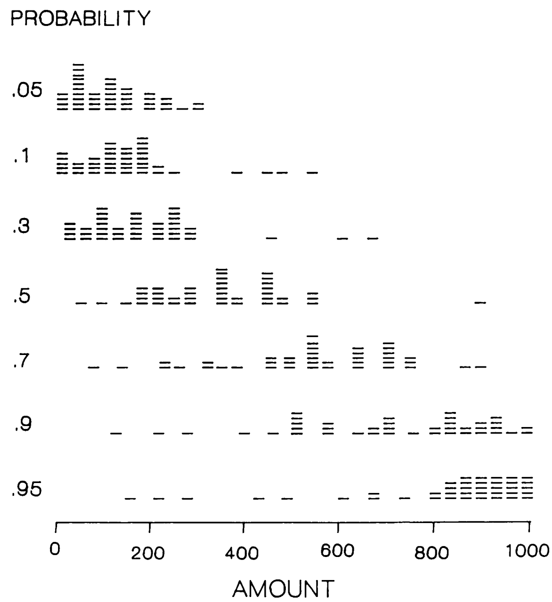

Fig. 3. Distributions of individual certain-immediate amounts equivalent to $1,000 with various probabilities.

The following instructions were read to all subjects in the probability discount groups:

> The purpose of this experiment is to compare your preferences for different amounts of money.
>
> In this experiment you will be asked to make a series of hypothetical decisions between monetary alternatives. The experimenter will present two sets of cards to you. The cards on your left will offer you an amount of money that will vary, but will always be given to you for sure. On the cards on your right the amount of money will be $1,000, but its payment will be uncertain. That is, there will be a specified chance that you get the $1,000. The chance of winning the $1,000 will be written as a percentage. Please ask the experimenter to show you an example of both sets of cards and clarify any questions you might have.
>
> You must always choose one of the two cards by pointing to it with your hand.
>
> Thanks for your cooperation.

Results and Discussion

Figure 3 shows, for all 40 subjects (pooled), distributions of amounts of money, available with certainty, between which, and $1,000

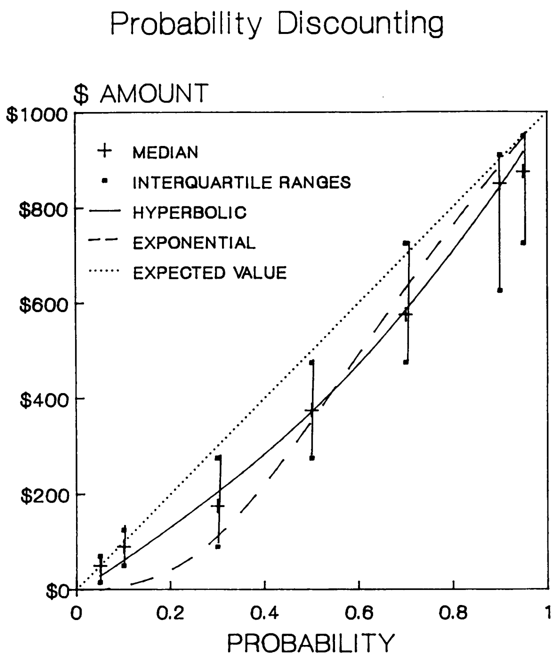

Fig. 4. Amount of certain-immediate money equivalent to $1,000 with various probabilities. The crosses are at the medians of individual subjects' points of indifference. Perpendicular bars (and small squares) represent interquartile ranges. The solid line is the best fitting hyperbolic function of odds against. The segmented line is the best fitting exponential function. The dotted line represents the expected value of $1,000.

available with the probability shown on the ordinate, subjects were indifferent. The distributions were highly skewed at the extremes but nearly normal at $p = .5$. Skewedness ranged from -1.46 at $p = .95$ to $+1.48$ at $p = .05$, with a minimum absolute value ($+0.27$) at $p = .5$. Kurtosis (a measure of peakedness) was also minimal at $p = .5$. Both the skewedness and kurtosis of these distributions can be explained by the limits imposed on the subjects' choices. An amount of money with a very high probability can never be worth more than the amount itself, and an amount with a very low probability can never be worth less than zero.

Figure 4 shows the medians and the interquartile ranges of the certain amounts of money subjectively equivalent to the probabilistic $1,000. The solid line in Figure 4 represents the hyperbolic equation:

$$v_p = \frac{1{,}000}{1 + 1.6\Theta} = \frac{1{,}000p}{1.6 - 0.6p} \qquad (7)$$

The solid line corresponds generally to the hypothetical function drawn by Kahneman and Tversky (1979) to explain apparent anomalies of human decision. Recall Allais' (1953) paradox that, with two probabilistic alternatives, further discounting of both alternatives by the same probability (q) may reverse subjective preferences. If $f(p) = p$ in Equation 1, such a reversal is impossible. But if $f(p)$ is as given by Equation 7, a reversal may occur. For instance, in Equation 7, the $1,000 reward with $p = 1$ is worth $1,000, but a $2,500 reward (substituting 2,500 for 1,000) with $p = .5$ would be worth only $962 (reflecting the commonly found "risk aversion"). Reducing both rewards by $q = .1$, the $1,000 reward would, according to Equation 7, be worth $65, whereas the $2,500 reward ($p = .05$) would be worth $80. Thus the hyperbolic probability discount function of Equation 7 resolves Allais' paradox and predicts preference reversals among probabilistic rewards just as the hyperbolic delay discount function of Equation 2a predicts preference reversals among delayed rewards.

The percentage of variance (r^2) explained by Equation 7 is .996 with a slope between predicted and obtained values of .987. For individual subjects, the median r^2 was .970 and the median slope was .963. Examples of some of the best individual fits are shown in Figure 5.

The dotted line of Figure 4 represents the expected value of the probabilistic alternative. Clearly the subjects' choices in this experiment deviated from this "rational" expectation. The segmented line of Figure 4 is the best fitting exponential function. The data also deviate from the exponential function. The best fitting exponential function tends to underdiscount the value of high-probability outcomes and to overdiscount the value of low-probability outcomes.

Figure 6 shows distributions of the amounts of money, available immediately, between which, and $1,000 available with delay shown on the ordinate, subjects were indifferent. There is a systematic pattern of deviations from normality in these distributions analogous to the one observed in the distributions for the probabilistic outcomes (Figure 3). Skewedness ranged from -2.04 at the 1-month delay to $+0.92$ at the 600-month delay, with a minimum absolute value of $+0.13$ at the 120-month delay. The minimum kurtosis ($+0.17$) appeared at the 12-month delay. Again, the lim-

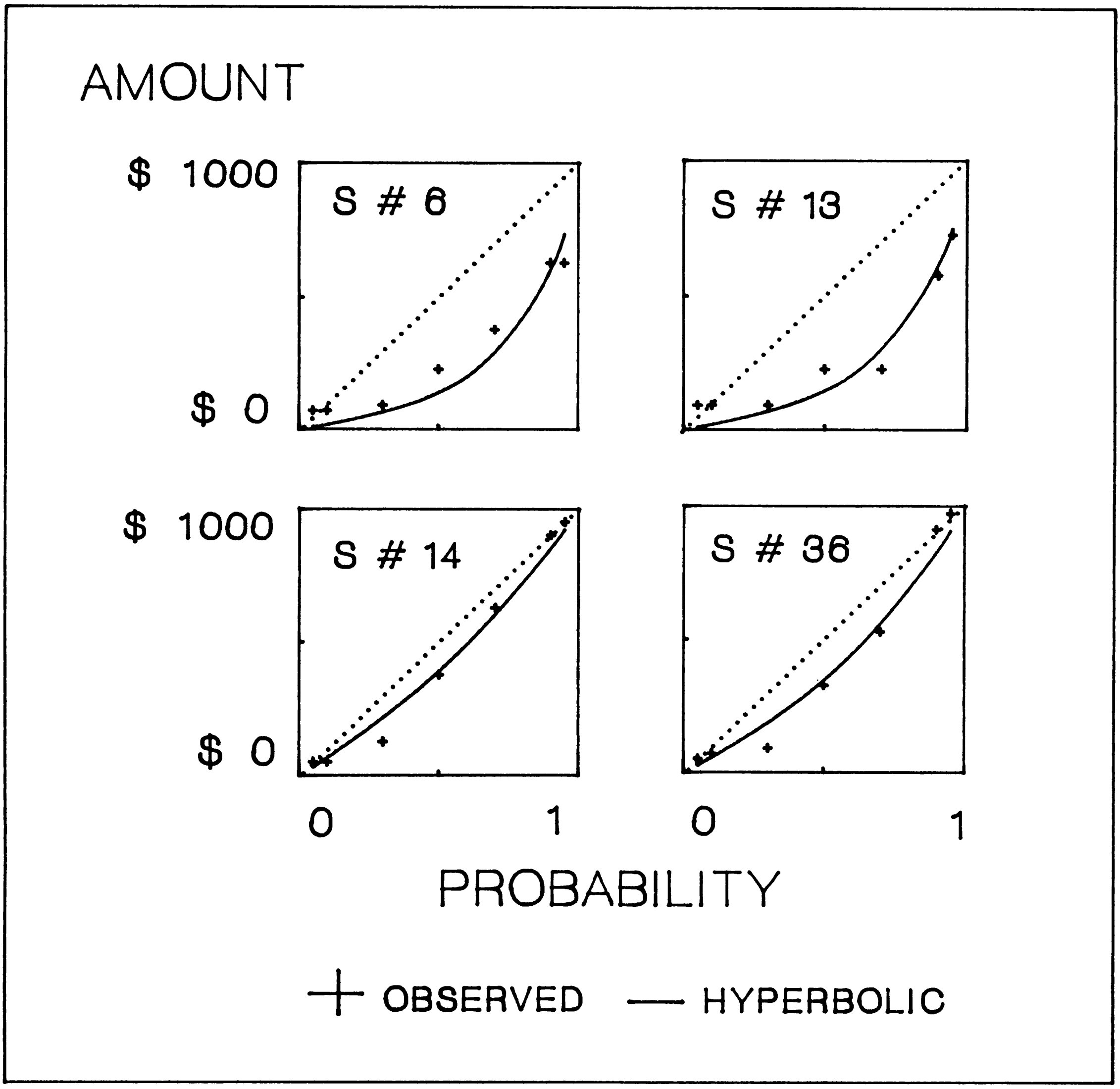

Fig. 5. Examples of some of the best fits of Equation 5 to individual probability discounts.

itations of the dependent variable (between \$1,000 and zero) constrained the distributions.

Figure 7 shows the medians and the interquartile ranges of the amounts of money considered by the subjects to be equivalent to the delayed \$1,000. The solid line in Figure 7 represents the following hyperbolic function:

$$v_d = \frac{1{,}000}{1 + 0.014d} \tag{8}$$

The percentage of variance (r^2) explained by Equation 8 is .995 with a slope of .953. For individual subjects, the median r^2 was .977 and the median slope was .993. Examples of some of the best individual fits are shown in Figure 8.

The segmented line in Figure 7 is the best fitting exponential function. As with probabilistic discounting, the exponential model tends to underdiscount the value of less delayed outcomes and to overdiscount the value of the more delayed outcomes when compared with the observed medians and the hyperbolic function.

The corresponding form of Equations 7 and 8 implies that odds against in probabilistic discounting acts like delay in delay discounting

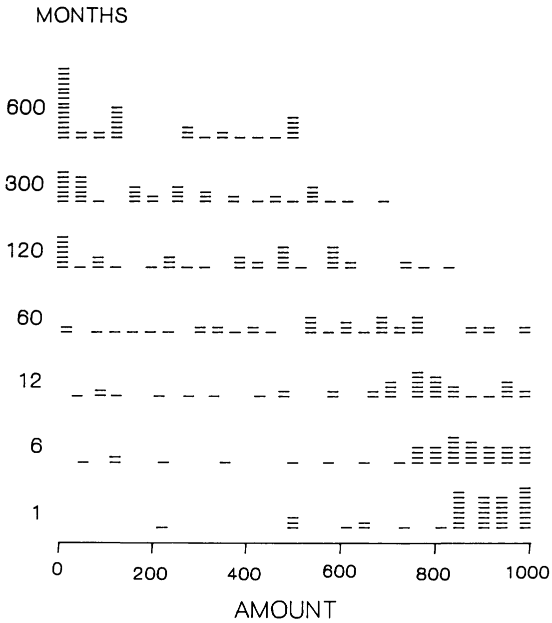

Fig. 6. Distributions of individual certain-immediate amounts equivalent to $1,000 with various delays.

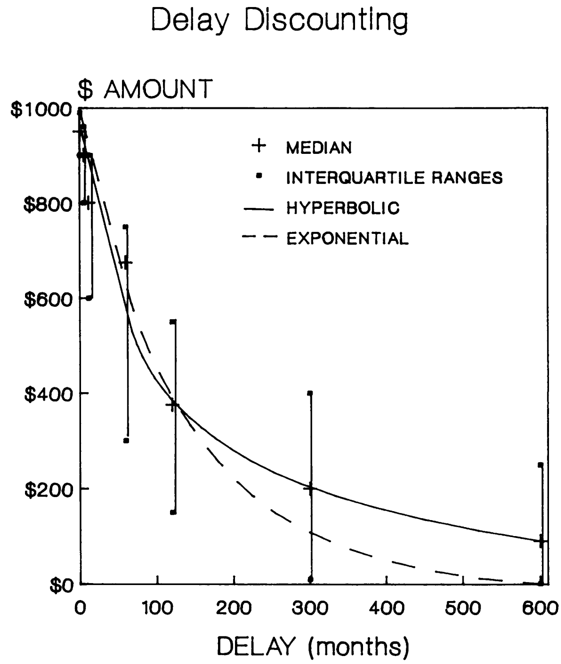

Fig. 7. Amount of certain-immediate money equivalent to $1,000 with various delays. The crosses are medians of individual subjects' points of indifference. Perpendicular bars (and small squares) represent interquartile ranges. The solid line is the best fitting hyperbolic function of delay. The segmented line is the best fitting exponential function.

and tends to confirm the speculation of Rachlin et al. (1986) that stated probability and stated delay have corresponding effects on behavior. As a further test of this speculation, another experiment was performed to find points of indifference between a probabilistic reward on the one hand and a delayed reward of equal amount on the other hand rather than (as in Experiment I) between a probabilistic or delayed reward on the one hand and an immediate-certain reward of varying amount on the other.

EXPERIMENT II

Setting the delay discount function given by Equation 2a equal to the probability discount function given by Equation 5 produces:

$$v_p = v_d$$

$$\frac{V}{1 + h\Theta} = \frac{V}{1 + kd}$$

$$d = (h/k)\Theta \qquad (9)$$

In the present experiment subjects chose between certain but delayed rewards and probabilistic but immediate rewards. If the probability and delay discount functions do indeed have the same form, Equation 9 should hold and a simple proportionality should be found between values of odds against (Θ) and delay that produce equivalent discounts of a $1,000 reward.

METHOD

Subjects

Forty students enrolled in an undergraduate psychology course at the State University of New York at Stony Brook served as subjects. Their participation was a course requirement.

Materials

Two sets of cards were presented in pairs to the subjects. One set of cards offered the subjects a probabilistic $1,000. The probabilities (again represented as percentages) were the same as those used in Experiment I. The other set of cards presented $1,000 to be obtained after a delay. The delay values used were 1 week, 1 month, 6 months, 1 year, 5 years, 10 years, 17 years, 25 years, 50 years, and 100 years.

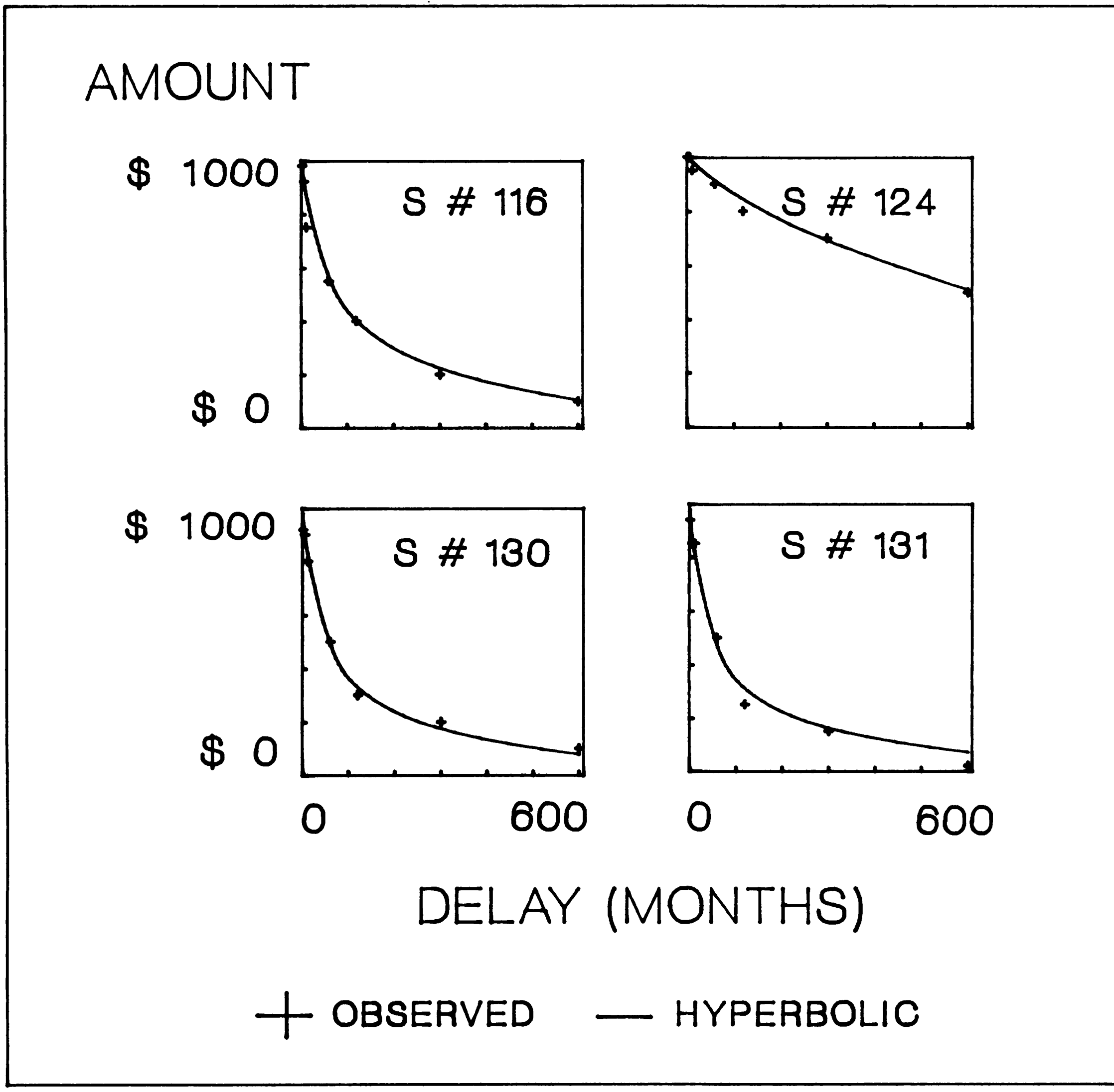

Fig. 8. Examples of some of the best fits of Equation 2a to individual delay discounts.

Procedure

The procedure was similar to that of Experiment I. Subjects were asked to state their preference between a card that represented a risky $1,000 versus a delayed $1,000. The probabilities were tested in descending order. With each probabilistic $1,000, 20 subjects were exposed first to a descending and then an ascending series of delays (20 to the reverse) until a delay was found equivalent to the probability.

Instructions were analogous to those in Experiment I.

Results and Discussion

Again distribution shapes (not shown for this experiment) reflect procedural constraints. At high probabilities, equivalent delays were constrained at zero and distributions are therefore skewed (at p = .95, skewedness = 3.46), but at low probabilities delays were relatively unconstrained (up to 100 years) and distributions tended to be more nearly normal (at p = .05, skewedness = 0.75).

Figure 9 shows the median equivalent delay (and interquartile ranges) as a function of probability. The dotted curve represents the

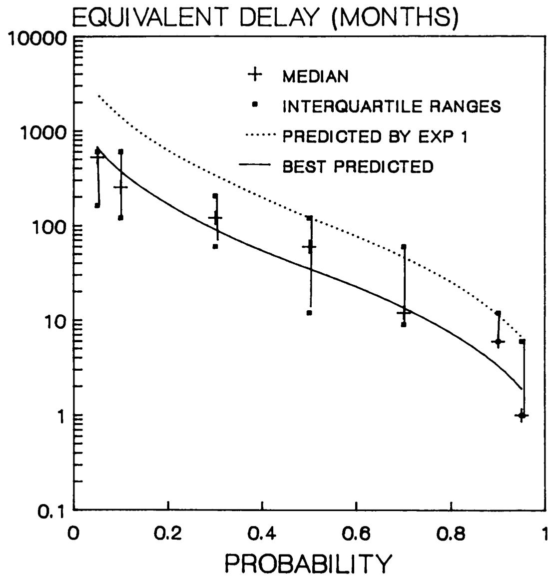

Fig. 9. Delay of $1,000 equivalent to $1,000 with various probabilities. The crosses are medians of individual subjects' points of indifference. Perpendicular bars represent interquartile ranges. The dotted line represents Equation 9 using parameter values obtained from Experiment I. The solid line is the best fitting linear function of odds against.

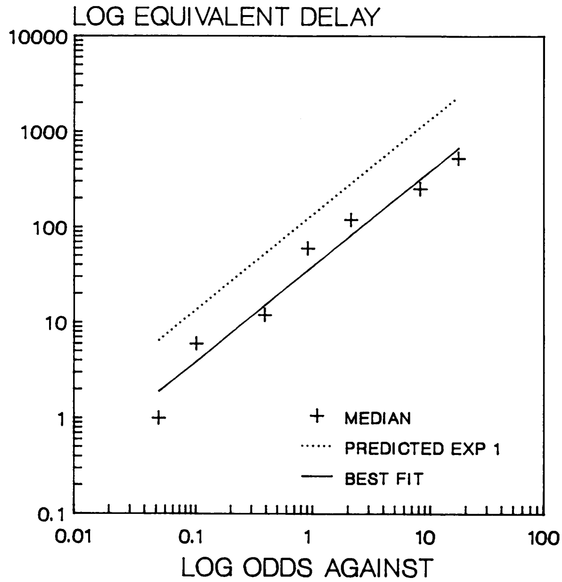

Fig. 10. Same data as Figure 9, now with the natural log of odds against rather than probability as the abscissa. Again, the dotted line represents Equation 9 using parameter values obtained from Experiment I, and the solid line is the best fitting linear function of odds against.

values predicted by Equation 9 with constraints h and k as obtained in Experiment I ($h = 1.6$; $k = 0.014$). The difference may be due to individual differences among subjects of the two experiments. More likely the discrepancy is due to procedural differences. In the two parts of Experiment I, probability and delay were both independent variables. In Experiment II, probability was the independent variable and delay was adjusted. Other experiments in which probability was adjusted (not reported here) resulted in a similar functional form that differed in the values of the constant.

The solid line of Figure 9 plots Equation 9 with the best fitting constant of proportionality ($h/k = 35.3$). Figure 10 is a log-log plot of delay versus odds against. Equation 9 predicts that the points on such a plot should form a straight line with a slope of 1.0. The percentage of variance explained by Equation 9 is .961 with a slope of .988. For individual subjects, the median r^2 was .868 and the median slope was .870.

An experiment corresponding to the present Experiment II was performed by Mazur (1987, Experiment 3) with pigeons that pecked keys to choose between a delayed but certain reward (2-s access to food) and an immediate but probabilistic reward of the same amount. As in the present experiment, delay was titrated up and down until a delay was found equivalent to a given probability. Unfortunately (for our purposes) only 4 subjects were tested with only five probabilities each (none greater than .5). Nevertheless, Equation 9 describes the results. The difference between Mazur's and the present results is mainly one of scale. In the present experiment, for instance, people said they were indifferent between $1,000 with a probability of .5 and that same sum for sure delayed by about 5 years. In Mazur's experiment, pigeons were indifferent between a 2-s access to mixed gain delivered with a probability of .5 and that same reward delayed by about 1 s.

Comparing Equation 9 with Equation 3a, the constant h/k occupies the place of the constant t, the interval between repeated probabilistic trials resulting in an expected waiting time d between the first trial and a positive outcome. With repeated probabilistic trials there is some debate whether variation of intertrial interval affects choice as Equation 3a predicts (Rachlin et al., 1986; Silberberg, Murray, Christiansen, & Asano, 1988). With nonrepeated (one-shot) trials, as in the present experiment, there is no clear objective correlate of this constant. The simple proportionality predicted by Equation 9 would hold for any pair of corresponding functions—linear, exponential or whatever—as well as hyperbolic. Strictly speaking, Experiment II tests whether odds against and delay are functionally analogous but not, as in Experiment I, what the function is. Note, however, that the linearity predicted by Equation 9 is between delay and odds against and not between delay and probability.

The significance of Equation 9 is not that it provides either a probability or a delay discount function. However, once either function is known, Equation 9 predicts the form of the other. The relationship is complex between delay and probability but is quite simple between delay and odds against. Thus, considered in terms of analogy to delay, odds against may be said to have a degree of behavioral meaningfulness not shared by probability.

It might be inferred from the present results that, for deciding among probabilistic outcomes, probabilities are better stated as odds against than as fractions. To test whether it is easier to judge the value of odds ratios than probabilistic rewards, we attempted to repeat Experiment II with stated odds against rather than stated probabilities. The subjects, again Stony Brook undergraduates not experienced in gambling, did not respond consistently; they could not understand the meaning of an odds ratio (e.g., "The odds are 3:1 against you . . .") well enough to judge delay of reward unless the odds ratio was accompanied by an equivalent probability (". . . which means that in four trials, you would win one time"). Experienced gamblers would undoubtedly have a better understanding of odds. However, it is unlikely that our subjects translated probabilities into odds against when they could understand the latter only in terms of the former.

It is therefore necessary to distinguish between the meaningfulness of a variable as it affects choice behavior and as it affects verbal understanding. Kahneman and Tversky (1979) make a corresponding distinction between "subjective probability" and "decision weight." Subjective probability represents human subjects' psychophysical *judgment* of verbally stated probability, but this judgment does not correspond to probability as it affects subjects' *decisions* between probabilistic alternatives. The relationship proposed by Kahneman and Tversky to apply between stated probability and decision weight corresponds over most of its range to that between probability and odds against.

GENERAL DISCUSSION

The generality of these results is limited. First, the experiments reported here used only one standard undiscounted value ($1,000) and therefore cannot account for interactions between amount of reward and degree of discounting. Second, the subjects' task in the present experiments was always to choose between alternatives rather than (as is more typical in studies of human discounting) to evaluate or judge a given alternative; value as measured by judgment procedures may differ significantly from value as measured by choice procedures (Bostic, Herrnstein, & Luce, in press). Third, the probabilities in the present experiments were stated as "one-shot" rather than repeated gambles. Keren and Wagenaar (1985) found significantly less underestimation of probability with choices among repeated gambles than with choices among one-shot probabilities. Generally, small differences in the framing or context of instruction may have large effects on human choice (Kahneman & Tversky, 1979; Mischel et al., 1989; Silberberg et al., 1988). Furthermore, the present experiments varied stated probability and delay separately. A single reward was either probabilistic or delayed but not both. Future experiments are necessary to study framing effects and combinatorial effects in both animal and human choice.

Nevertheless, several regularities emerged within the context of the present procedure. In Experiment I, the hyperbolic delay discount function found by Mazur (1987) to describe the behavior of food-deprived pigeon subjects choosing between delayed food rewards of different amounts also described the behavior of human subjects choosing between delayed and immediate hypothetical monetary rewards of different amounts. Furthermore, with a transformation of probability to equivalent waiting time (odds against), the same discount function described the behavior of human subjects choosing between probabilistic and certain hypothetical monetary rewards of different amounts. Experiment II showed that human subjects could directly translate between probability and equivalent delay of a monetary reward of fixed amount.

These results confirm previous findings by Benzion et al. (1989), Mischel and Grusec (1967), Rachlin et al. (1986), and Stevenson (1986) that probability and delay discounting affect human behavior in corresponding ways. The present results suggest further that the specific form of the delay discount function for humans is the same as that of pigeons and that the form of the human probability discount function is derivable from that of the delay discount function. The (hyperbolic) shape of this function accounts for preference reversals between delayed rewards and has been used to describe pigeon self-control (or the lack of it) and commitment (Rachlin & Green, 1972). The present experiment lends some support to the extension of such considerations to human self-control and commitment (Logue, 1988). The same hyperbolic shape, in the form of waiting times equivalent to stated probabilities, has been shown to explain corresponding reversals of human decisions among probabilistic rewards (Rachlin et al., 1986).

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